# AN681

# Reading and Using Fast Fourier Transforms (FFT)


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## INTRODUCTION

When you're trying to solve a signal integrity problem, the best of all worlds is to have more than one tool to examine the behavior of a system. If there is an Analog-to-Digital (A/D) converter in the signal path, there are three fundamental issues that can be examined when assessing the circuit's performance. All three of these methods evaluate the conversion process as well as its interaction with other portions of the circuit. All data collected for this application note used MXDEV™ hardware and MXLAB software development tools. These three areas of concern encompass the use of frequency analysis (FFTs), time analysis, and DC analysis techniques.

DC or static results are used to ascertain the accuracy of the system. The evaluation of offset errors and gain errors render insight into the cause of signal clipping and absolute measurement errors. Investigations into the differential linearity and integral linearity of the system provide hints about the level of signal distortion over the signal's full scale range.

On the other hand, the combination of the time and frequency analysis reveal the rest of the story. An examination of time domain data can quickly show systematic problems, such as signal modulation or drift over time. But when the time domain data does not identify the problem, evaluations in the frequency domain can bring out fairly significant issues.

A frequency domain evaluation can quickly illustrate the influence of noise sources, line frequency or the inadequacies of the analog and digital electronics in the signal path.

The Fast Fourier Transform (FFT) tool is used to evaluate the AC performance of these types of digitizing systems in the frequency domain. The theory of the Fourier series is somewhat complex, but the application is simple. The Fourier transform operates on the premise that any signal or waveform can be reconstructed by just adding together one or more pure sine waves with their appropriate amplitude, frequency, and phase.

For example, a square wave can be constructed from the Fourier series, sin(x) + 1/3 sin(3x) + 1/5 sin(5x) + 1/7 sin(7x)... With the addition of each element of this series, the fundamental pure sine wave (sin(x)) begins to transform into a square wave as illustrated in Figure 1. The result of this mathematical simplification can then be graphically illustrated, where the x-axis is shown in hertz and the y-axis in decibels.

An FFT is a computational fast way to calculate the Discrete Fourier Transform (DFT). The DFT is a discrete numerical equivalent of the signal using sums. The elements of these sums are more generalized, in that the relationship between the harmonics are removed. In this manner, any signal can be evaluated.

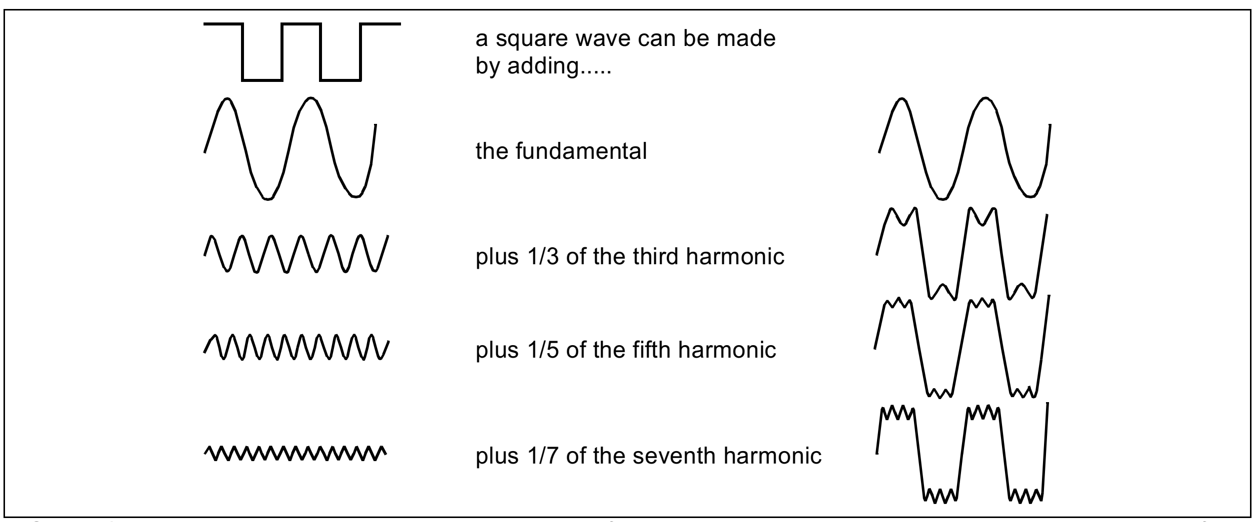
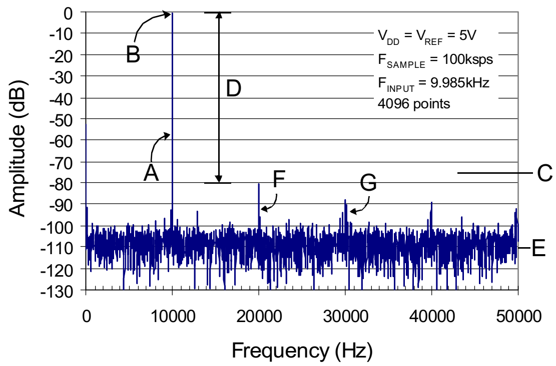



***FIGURE 1:*** *A square wave can be constructed using a fundamental sine wave and adding the odd harmonics of that sine wave*




A: Fundamental Input Signal Magnitude
B: Headroom = -0.5 db
C: Signal-to-Noise Ratio = 72 db
D: Spurious Free Dynamic Range = 78.5 db
E: Average Noise Floor = -107 db
F: First Harmonic Magnitude = -79 db
G: Second Harmonic Magnitude = -89 db

***FIGURE 2:*** *Basic elements of the FFT plot include: the fundamental input signal (A), signal headroom (B), signal-to-noise ratio (C), spurious-free dynamic range (D), and the average noise floor (E).*

## READING THE FFT PLOT

An FFT plot is generated by collecting a large number of digital samples from the output of the A/D in a periodic fashion. Typically, A/D converter manufacturers use a single tone, full-scale analog signal at the input of the A/D converter for their typical performance curves that are provided in the specification sheets. Under these conditions the full dynamic range of the converter is exercised. This data is then converted to a plot similar to the one shown in Figure 2. The frequency scale of this plot is always linear and ranges from zero to nyquist/2. With FFT plots the nyquist frequency is equal to the sampling frequency of the converter. The time domain plot for the input signal to the A/D converter is shown in Figure 3.

The magnitude axis (y-axis) ranges from zero down to an appropriate negative value, depending the number of converter bits and the number of samples included in the FFT calculation. When an analog input signal generates a full-scale output from the A/D converter, it will appear as zero dB on the FFT plot. Any magnitude less than full-scale can easily be converted into the digital code representation with these formulas:

$$D_{OUT} = (2^n - 1) \times 10^{(MAGNITUDE/20)}$$

$$V_{OUT}(RTI) = D_{OUT} \times FSR/2^n$$

Where:

$D_{OUT}$ = a decimal representation of the digital output code. $D_{OUT}$ should be rounded to the nearest integer

*MAGNITUDE* = taken from the FFT plot and is in dB

$V_{OUT}$ *(RTI)* = a mathematical calculation that converts $D_{OUT}$ into the same units as the analog input voltage.

*RTI* = Referred To Input. This number should approximate the analog input voltage, $V_{IN}$.

*n* = the number of A/D converter bits

*FSR* is the analog full-scale input range in volts

There are five elements shown in Figure 2 of particular interest in the FFT plot that provide insight into the system performance.

### Fundamental Input Signal

The FFT plot in Figure 2 was generated using the MCP3201, a 12-bit A/D converter. The MCP3201 is configured for a sampling frequency of 75 kHz with a clock rate of 1.2 MHz. The analog input signal frequency is 36 kHz (Figure 2.A). A total of 4096 12-bit words were taken from the MCP3201 to generate this plot.

### Input Signal Headroom

In reference to Figure 2, the highest spur (A) represents the fundamental input signal to the MCP3201. This signal is used to exercise the converter's codes. In this case the input signal is exercising the converter over as much of its input range as possible. The amplitude of the fundamental frequency in Figure 2 is 0.5 dB or 94.4% lower than full-scale, giving Headroom (B) for the converter's output. This is done to ensure that the converter is not overdriven, which will cause signal clipping. If signal clipping occurs the FFT plot will show distortion of that signal in the form of spurs at frequencies other than the fundamental frequency.

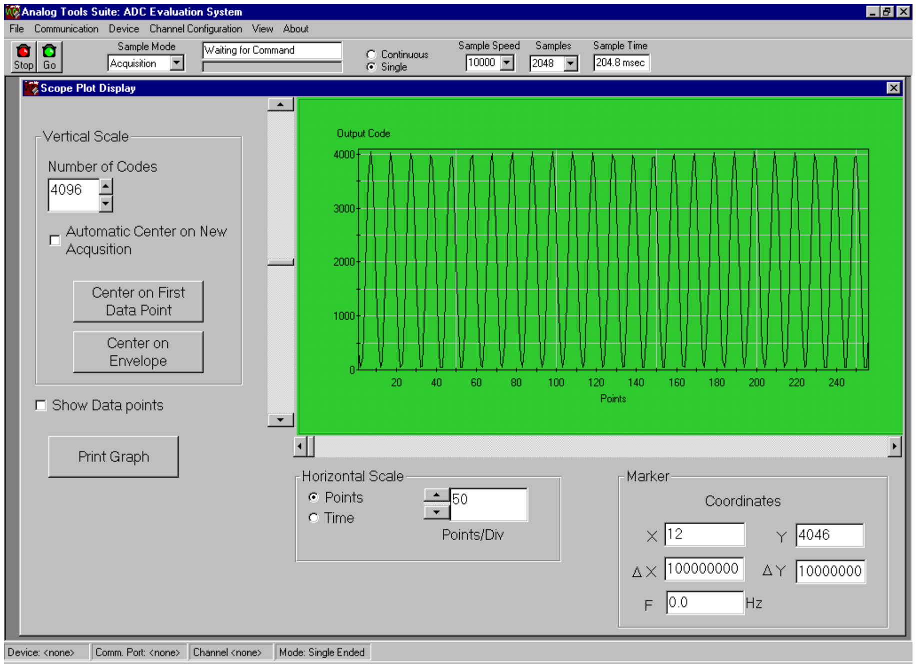



***FIGURE 3:*** *The FFT plot is constructed with 2048 samples. This time domain signal was converted by the MCP3201. The digital output of this conversion was used to create the FFT plot in* Figure 2.

## SIGNAL-TO-NOISE RATIO

A useful way of determining noise in an A/D converter circuit is with the Signal-to-Noise Ratio (C). The Signal-to-Noise Ratio (SNR) is a calculated value. It is the ratio of signal power to noise power. The theoretical limit of SNR is equal to 6.02n + 1.76 dB, where n is the number of bits. An ideal 12-bit A/D converter would have a SNR of 74 dB. All spurs and the noise floor are included in the SNR FFT calculation.

$$\begin{aligned} SNR &= (rms\ signal)/(rms\ noise) \\ &= (LSB2^{n-1}/\sqrt{2})/(LSB\sqrt{12}) \\ &= (6.02n + 1.76dB) \end{aligned}$$

Where:

*LSB* is the voltage width of the least significant bit

The SNR of the FFT calculation is a combination of several noise sources. The possible noise sources include the quantization error of the A/D converter, internal noise of the A/D converter, noise from the voltage reference, differential nonlinearity errors from the A/D converter and noise from the driving amplifier.

### Spurious-Free Dynamic Range

The Spurious-Free Dynamic Range (D) quantifies the amount of distortion in the system. The Spurious-Free Dynamic Range (SFDR) is defined as the distance from the fundamental input signal to the first spur (in dB).

Spurs resulting from the nonlinearity of the A/D converter will appear as a multiple (B, E, F, G) of the input signal's frequency (A), i.e. Asin(bx), unless they are a result of aliasing. If the spurs are a result of the aliasing phenomena they are equal to:

$$f_{INTERFERENCE} = Kf_{SAMPLE} - f_{ALIASED}$$

Where:

$f_{INTERFERENCE}$ is the calculated possibilities of high frequency interference

*K* is a positive whole number

$f_{SAMPLE}$ is the sampling frequency of the A/D converter

$f_{ALIASED}$ is the aliased signal that appears on the FFT graph

In general, harmonically related spurs are caused by errors in the A/D converter. Non-harmonically related spurs are a result of other devices or external noise sources.

If spurs are created by the A/D converter, it is probable that the converter has a degree of integral nonlinearity. These spurs can also be created by the signal source or the driving amplifier, in which case, the frequency of these spurs are not related to the frequency of the fundamental frequency per the formula above. If the driving amplifier is the culprit it may have cross over distortion, be unable to drive the A/D converter, or be bandwidth limited. Spurs can also be caused by injected noise from other places in the circuit, such as digital clock sources or the mains frequency.

### Average Noise Floor

The average noise floor (E) in Figure 2 is a combination of the number bits and the number of points used in the FFT. It is not a reflection of the performance of the A/D converter. Regardless of the number of bits that the A/D converter has, the number of samples should be chosen so that the noise floor is below any spurs of interest.

$$AverageFFTNoiseFloor(dB) = 6.02n + 1.76dB \\ + 10\log(3 \times M / (\pi \times ENBW))$$

where:

$M$ = the number of data points in the FFT

$ENBW$ = the equivalent noise bandwidth of the window function

$n$ = the number of bits of the A/D converter

A reasonable number of samples for the FFT of a 12-bit converter is 4096.

There are two other specification of interest that the FFT calculations produce; Total Harmonic Distortion (THD) and Signal-to-Noise plus Distortion (SINAD). THD is the rms sum of the powers of the harmonic components (spurs) ratioed to the input signal power. Refer to Equation 1 for formula.

Significant integral nonlinearity errors of the A/D converter typically appear in the THD results. Microchip specifies THD to include the first five harmonic components in this calculation.

SINAD is a calculated combination of SNR and THD, where:

$$SINAD = -20\log\sqrt{\left(10^{-SNR/10} + 10^{+THD/10}\right)}$$

$$THD_{RMS} = 20\log\sqrt{\left(10^{2ndHAR/20}\right)^2 + \left(10^{3rdHAR/20}\right)^2 + \left(10^{4thHAR/20}\right)^2 + (\ldots)}$$

**EQUATION 1: FORMULA FOR CALCULATING TOTAL HARMONIC DISTORTION (THD)**

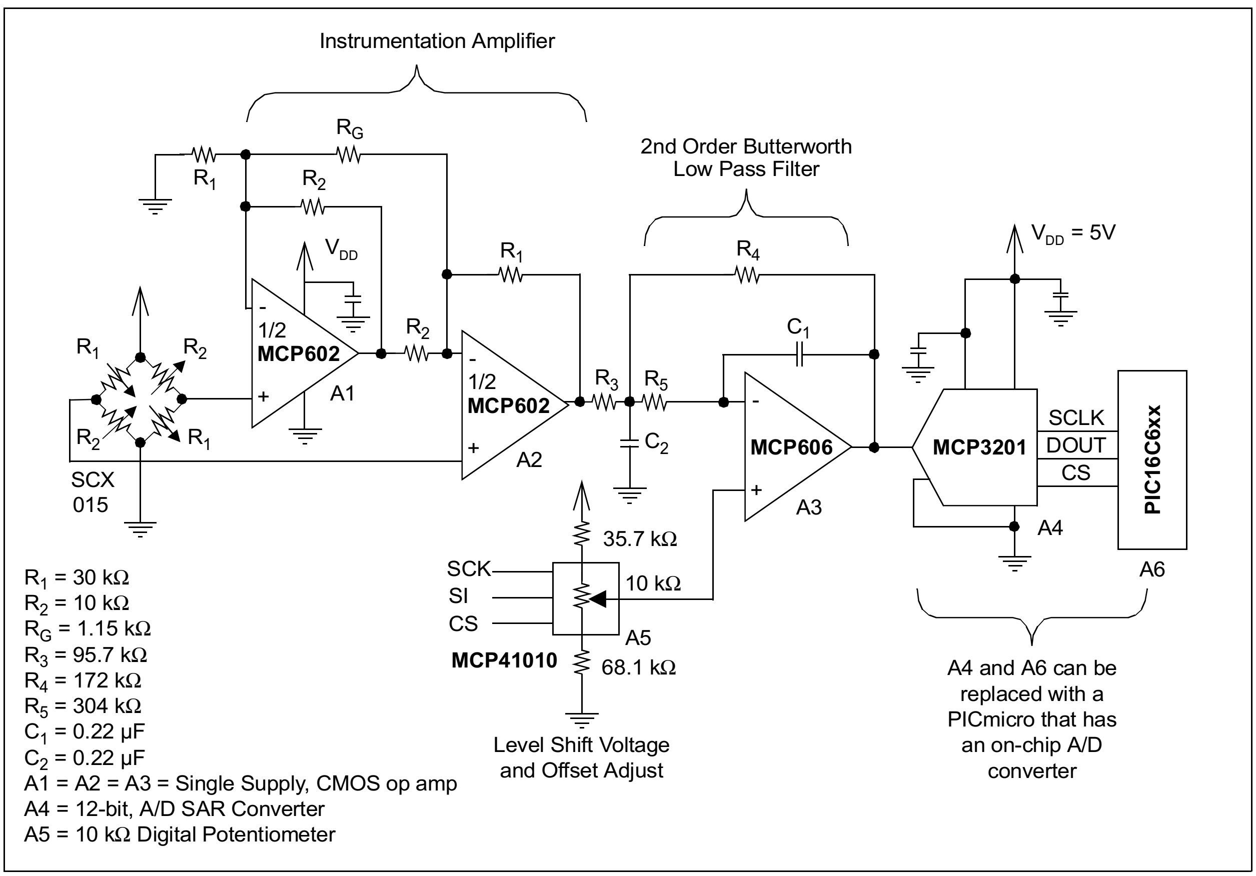



***FIGURE 4:*** *The voltage at the output of the SCX015 pressure sensor is gained by the instrumentation amplifier (A1 and A2) then filtered, gained and level shifted (A5) with a 2nd order low pass filter (A3) and digitized with a 12-bit A/D converter (A4).*

## PROBLEM SOLVING WITH FFTS

A typical pressure sensing analog signal path using the 12-bit A/D converter, MCP3201 is shown in Figure 4. In this example, a pressure sensor (SCX015) arranged in a resistive bridge configuration is powered by the supply voltage, $V_{DD}$. This excitation generates a small differential voltage across the output terminals of the sensor. The variation in differential voltage at the output of this sensor is equal to a nominal 6.0mV/psi. The instrumentation amplifier in this circuit (comprising of A1 and A2) gains the differential signal from the output of the sensor and sends that difference to the second order low pass filter, A3. The signal is then transmitted to the 12-bit A/D converter, A4, which digitizes the signal in preparation for transmission to the microcontroller, A6.

### Power Supply Noise

A common source of interference in circuit applications is from the power supply. This interference signal is typically injected through the power supply pins of the active devices. For instance, an FFT plot of the output of the circuit shown in Figure 4 is given in Figure 5. In this case the instrumentation amplifier, voltage reference and A/D converter do not have bypass capacitors installed. Additionally, the inputs to the circuit are both referenced to a low noise, DC voltage source of 2.5V. Consequently, there is not a fundamental input frequency on the FFT plot. Note that there are other signals with harmonic content that are captured in the plot.

Further investigation into the circuit shows that the source of the noise seen on the FFT plot comes from the switching power supply. An inductive choke is added to the circuit along with bypass capacitors. One capacitor, 10 µF is positioned at the power supply and three 0.1 µF capacitors are placed as close the supply pins of the active elements as possible. The histogram results (Figure 7) of this change to the circuit indicates that the noise source has been successfully found and eliminated from the signal path of the circuit. A histogram representation of the data was used because FFT plots can not be generated with one code.

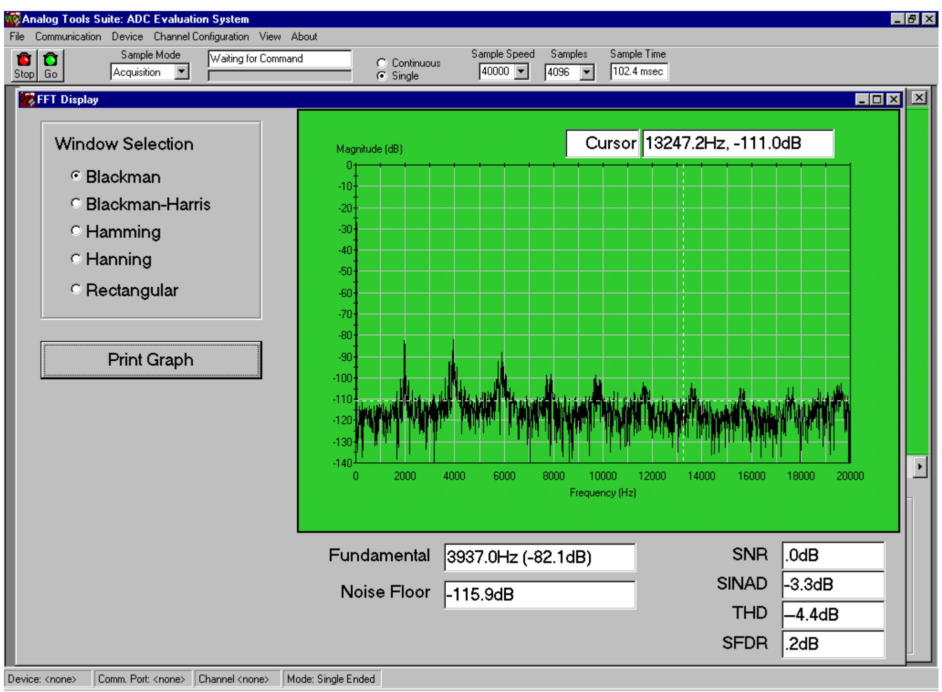



*FIGURE 5: Using the circuit in Figure 4, noise is injected into the power supply. The data for this FFT plot was taken from the MCP3201 A/D converter.*

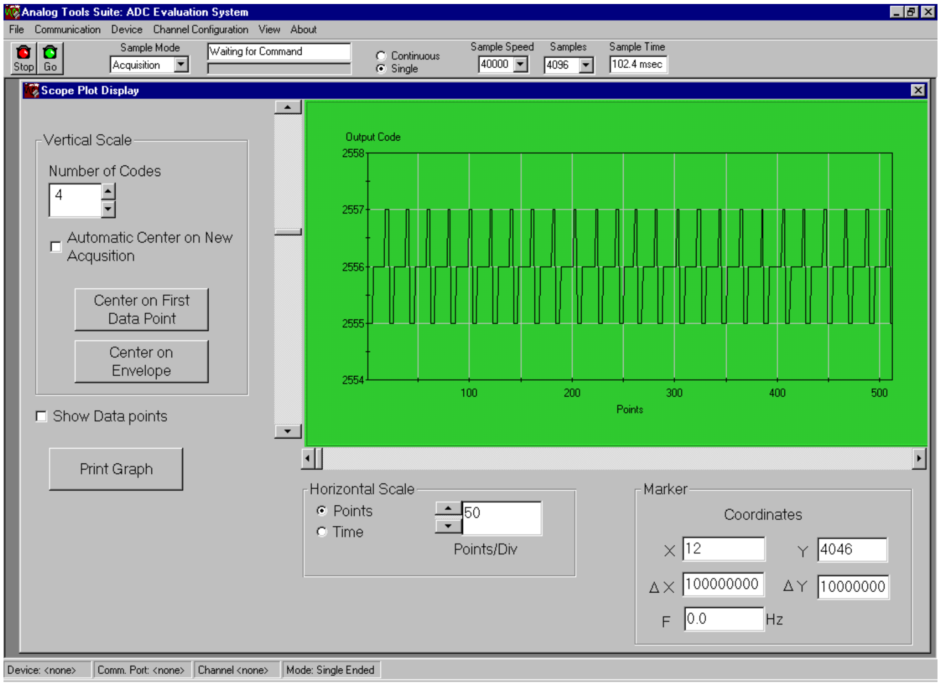



***FIGURE 6:*** *The noise that was injected into the circuit to generate the FFT plot in Figure 5 is shown in the time domain in this figure.*

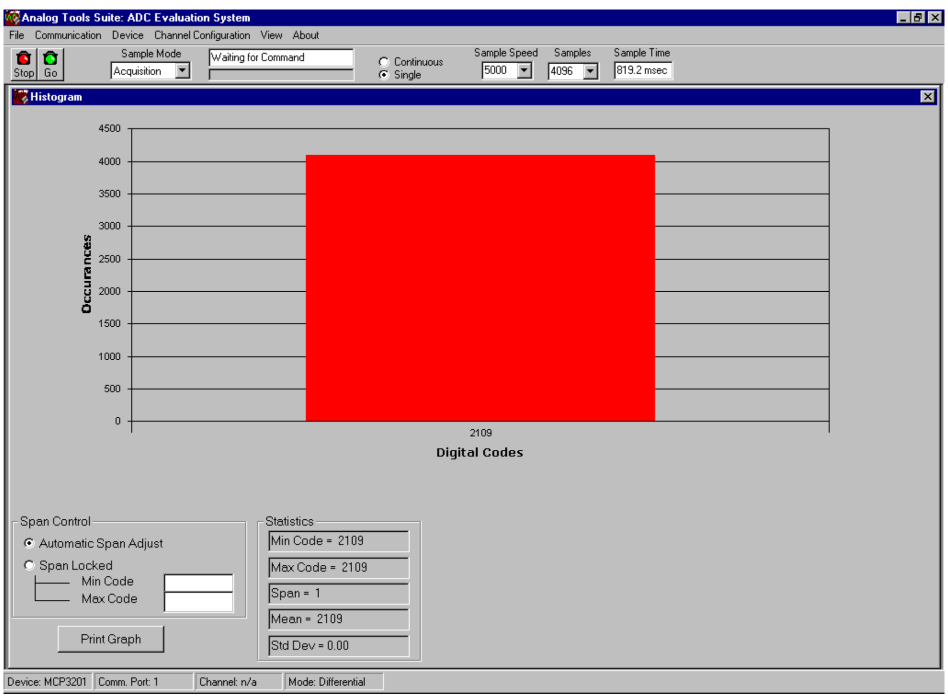



***FIGURE 7:*** *The noise measured in the FFT plot in Figure 5 is eliminated with an inductive choke on the analog power supply and bypass capacitors.*

### Interfering External Clocks

Another source of systematic noise can come from clock sources or digital switching in the circuit. If this type of noise is correlated with the conversion process, it will not appear as interference in the conversion process. However, if it is uncorrelated, it can easily be found with an FFT analysis.

An example of clocking signal interference is shown in the FFT plot in Figure 8. With this plot, the circuit shown in Figure 4 is used with the bypass capacitors installed. The spurs seen in the FFT plot shown in Figure 8 are generated by a 19.84 MHz clock signal on the board. In this instance, layout has been done with little regard for trace to trace coupling. The negligence to this detail appears in the FFT plot.

This problem can be solved by changing the layout to keep high impedance analog traces away from digital switching traces or implementing an anti-aliasing filter in the analog signal path prior to the A/D converter.

Random trace to trace coupling is somewhat more difficult to find. In these instances, time domain analysis can be more productive.

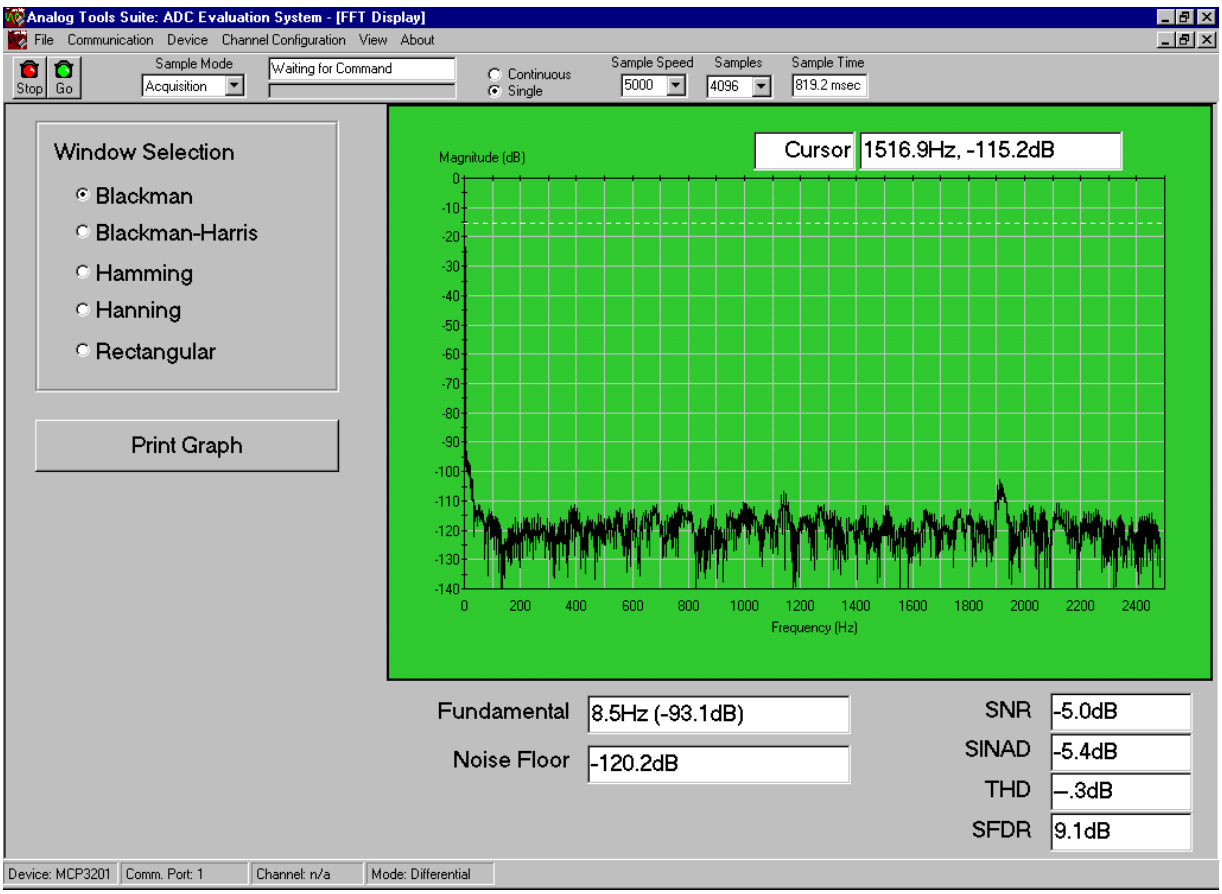



***FIGURE 8:*** *This FFT plot shows that clock noise from the digital portion of the circuit has injected into the analog signal path.*

### Improper Use of Amplifiers

Returning to the circuit shown in Figure 4, a 1 kHz AC signal is injected at the positive input to the instrumentation amplifier. This signal would not be characteristic of the pressure sensing circuit shown in Figure 2, however, this example is used to illustrate the influence of other devices in the analog signal path.

The performance of this circuit with the above conditions is shown in the FFT plot in Figure 9. It should be noticed that the fundamental seems distorted and there are numerous harmonics.

This distortion is caused by overdriving the amplifier slightly to the rail. The solution is to lower the amplifier gain to have a wider bandwidth and more current drive.

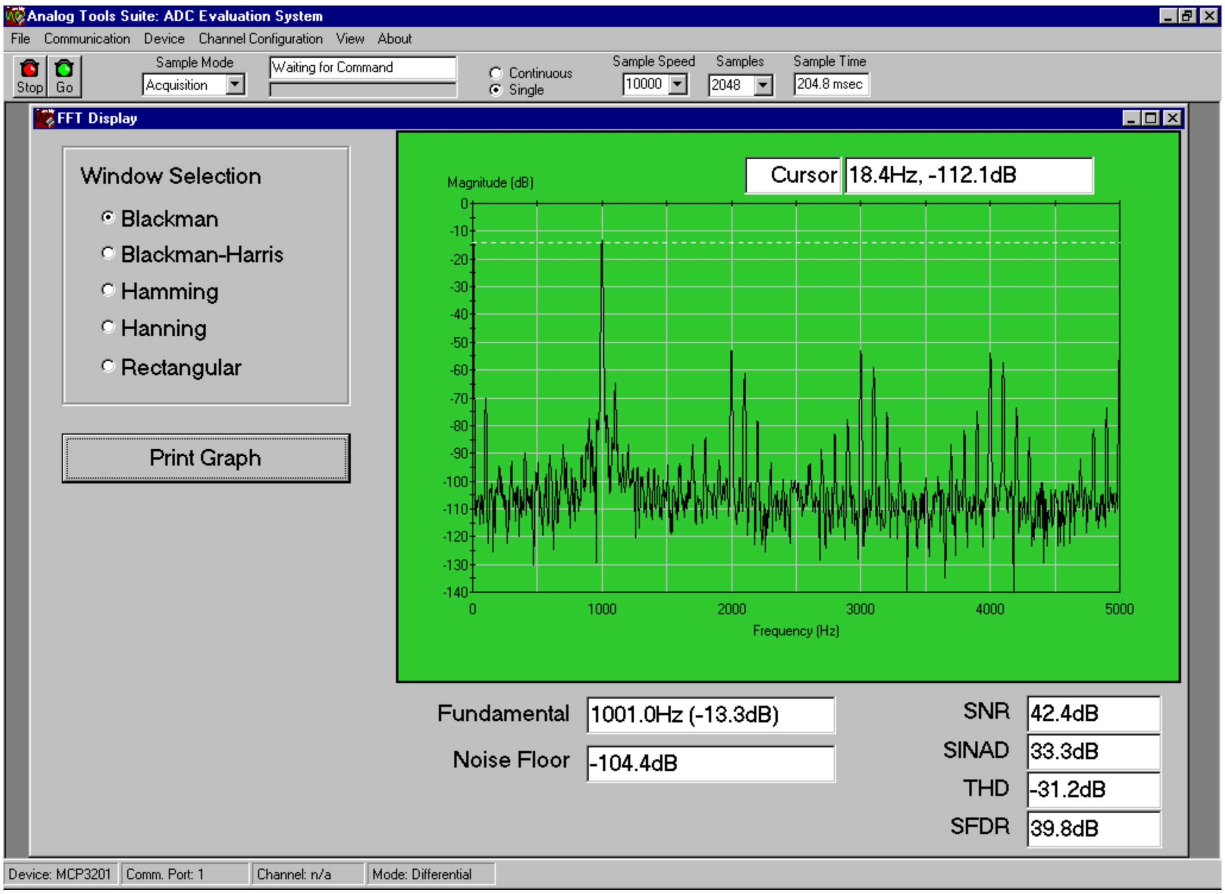



***FIGURE 9:*** *The driving amplifier to the A/D converter of the circuit in Figure 4 creates distortion as shown in this FFT plot.*

### Ambient Line Noise

In many industrial environments, emitted noise can be coupled into a circuit at various points. A common place for this to occur is with differential lines that are improperly implemented. In this example, the two-foot wires to the pressure sensor are routed down different paths. The ambient noise, in this case lines voltage is quickly pick up by these wires. The FFT plot in Figure 10 shows several spurs at frequencies that are multiples of 60 Hz.

This problem is easily corrected by twisting the wire pair to the sensor.

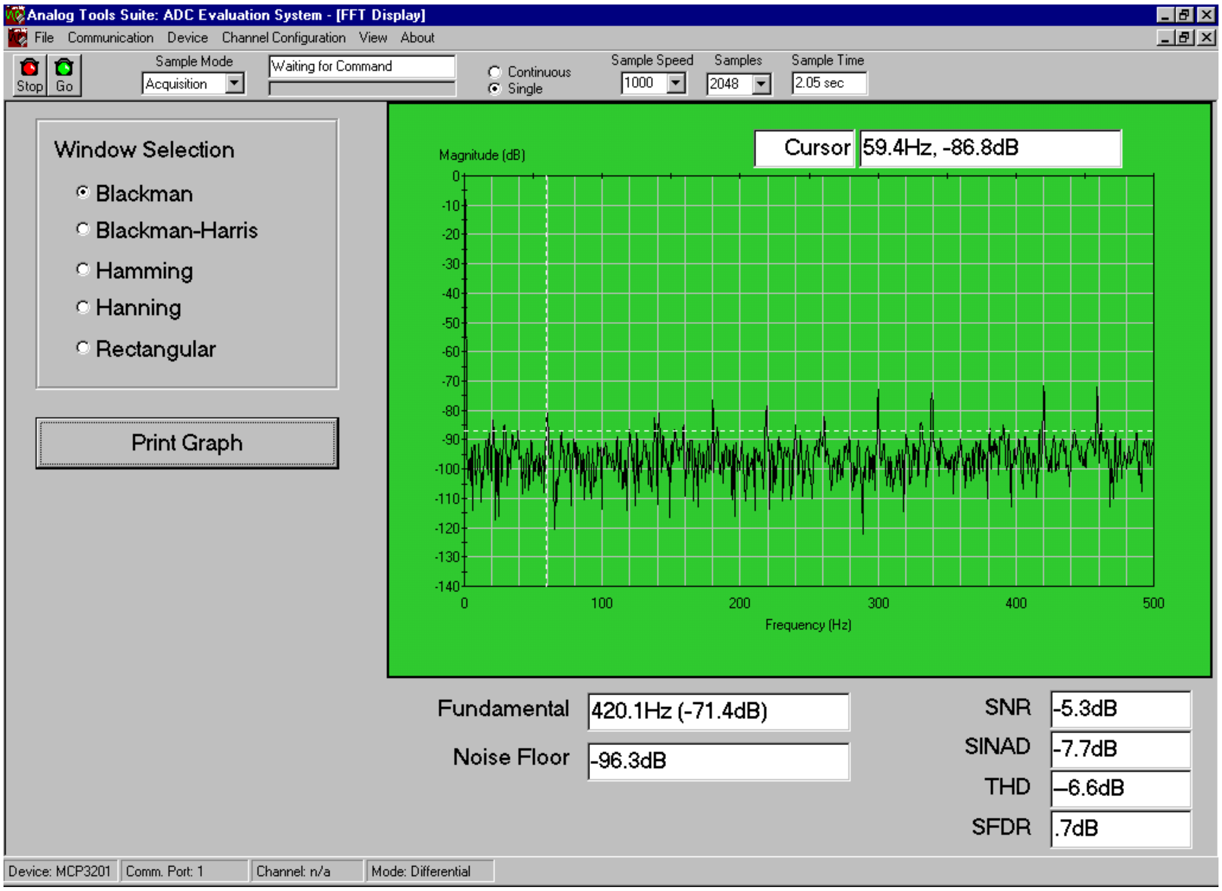



***FIGURE 10:*** *This FFT plot shows the affects of ambient noise on the circuit shown in Figure 4.*

## CONCLUSION

A/D converter manufacturers use FFT plots to illustrate the performance of their A/D converters. The information contained in these plots provide useful information while selecting the right converter for the application, but it also provides a starting point when troubleshooting the overall embedded application.

FFTs can help identify noise interference, power supply noise problems and analog device performance.

On the other hand, FFTs cannot provide all the troubleshooting information where time and DC analysis tools should be resorted to.

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